

Name: _____

Date: _____

Math 12H: Section 4.6 Basic Trigonometric Identities

$$\sin(-x) = -\sin x \quad \tan x = \frac{\sin x}{\cos x} \quad \csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

$$\cos(-x) = \cos x \quad \sin^2 x + \cos^2 x = 1 \quad \sin 2x = 2 \sin x \cos x$$

1. **Verify** which of the following are trigonometric identities:

a) $\tan x + \cot x = \sec x \csc x$	b) $\sec^2 x + \csc^2 x = \sec^2 x \cdot \csc^2 x$
c) $\sec^2 x - \csc^2 x = \frac{\sec^2 x}{\csc^2 x}$	d) $\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$
e) $\cos^2 x = \sin x (\csc x + \sin x)$	f) $\sin^2 x = \cos x (\sec x - \cos x)$
g) $\sin x \tan x + \sec x = \frac{\sin^2 x + 1}{\cos x}$	h) $\frac{\sin x + \tan x}{\cos x + 1} = \tan x$

2. Simplify each of the following expressions:

a) $\sin^2 x + \cos^2 x + \cot^2 x$	b) $\frac{\sin 2x}{1 + \cos 2x}$
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c) $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$	d) $\cos(a+b)\cos b + \sin(a+b)\sin b$
e) $\sin\left(\frac{\pi}{3} - x\right)\cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right)\sin\left(\frac{\pi}{3} + x\right)$	f) $\frac{\cos^3 x - \cos x}{\sin^3 x}$

3. Prove the following Identities:

a) $\sec \theta - \tan \theta \sin \theta = \frac{1}{\sec \theta}$	b) $\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$
c) $\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$	d) $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$

$$e) \frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$$

$$f) \tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta$$

4. Indicate whether if the following is an EVEN or ODD function: Justify your choice:

i) $\cos \theta$

ii) $\sin \theta$

iii) $\tan \theta$

iv) $\sec \theta$

v) $\cot \theta$

vi) $\sin \theta \cos \theta$

vii) $\cos \theta + 1$

viii) $3 \sin \left(\theta - \frac{\pi}{2} \right)$

ix) $2 \sin \theta \cos \theta$

5. Given that $\cos \theta = \frac{a}{b}$, then what is the value of $\cos(-\theta)$?

6. Prove the following identities:

a) $(\csc \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

b) $\sin^2 \theta \tan \theta + \cos^2 \theta \cot \theta + 2 \sin \theta \cos \theta = \tan \theta + \cot \theta$

c) $\sec^6 \theta - \tan^6 \theta = 1 + 3 \tan^2 \theta \sec^2 \theta$

d) $\frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta + \cos \theta} = \frac{\csc^2 \theta - \cot \theta - 2 \cos^2 \theta}{1 - \cot^2 \theta}$